THEORY OF JOINT LIABILITIES, ADVERSE SELECTION, ASSORTATIVE MATCHING AND SELF-FINANCING
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Abstract:
Microfinance is seen to be a remedy of poverty eradication and globally it is perceived that microfinance can remove the problem of poverty. Basically microfinance works on joint liability model. Traditional theories of credit lending say that rural credit markets are imperfectly competitive and acquiring information about borrowers type that is who is risky and who is safe is not costless. This market imperfection leads to high interest rate and drives out safe borrower from the credit market. In economic literature this problem is considered as adverse selection problem. Joint liability model try to solve the problem of adverse selection through group lending. This paper explores the idea of joint liability model and tries to solve the problem of adverse selection through the positive assortative matching. Paper concludes that in positive assortative matching, the payoffs of borrowers would be more than the payoffs of negative assortative matching. Paper, also try to show that self financing can bring down the interest rate and size of penalty and improve the borrower’s expected payoffs.

Keywords: adverse selection, assortative matching, credit market imperfection, joint liability, payoffs.

1. Introduction
In this paper, we look at the problem of private information. The potential borrowers are social connected and live in an informational permissive environment, where they know themselves and each other well. The lender does not know of this information domain and thus does not have access to the borrower’s information pool. Then lender can use contracts to solve this problem (Maurya, R, 2010). The paper explore specific type of contract which bind people together in groups, and allowing the lenders to solve the informational problem from borrower’s social network.

The potential borrowers differ in their ability to successfully run their project. The credit market contract lending, any information about the borrowers that affects the outcome is considered the type of the borrower. This type of information, which the lender would like to acquire before he lends to the borrower. The problem of acquiring types of borrower information is to referred to as the adverse selection problem in contract lending theory.

2. Why Group Lending?
Poor individuals lack formal credit because lenders have little means of screening clients, monitoring the use of funds, or enforcing repayment. In recent years, many development organizations have used group lending to deliver credit to poor individuals. Group lending aims to pass off the screening, monitoring and enforcement of loans to the peers (Banerjee et al. 1994, Diamond, 1984; Ghatak and Guinnane, 1999; Stiglitz, 1993; Varian, 1990, Karlan, 2007). Furthermore, group loans help formal lenders overcomes the prohibitively high fixed cost of delivering small loans.

Group lending mechanisms provide incentives to the borrowers to monitor each other to see who can pay and who cannot. Armendariz de Aghion and Gollier (2000) and Armendariz de Aghion (1999) shows theoretically how peer monitoring alone, with random formation of groups, can help overcome adverse selection problems when monitoring is costly for lending institution itself. Strong social networks have lower monitoring cost, which results in more credit being extended. To enforce lending contracts, lending institutions typically resort to legal options, such as seizing property of the borrower or garnishing wages directly from the employer. In most poor
countries, such punishments fail for one of the two reasons, either the legal infrastructure does not support such action, or the borrower has no sizeable assets or wages. De Soto (2000) and Besley and Coate (1995) discuss these issues at length. Group lending purports to overcome these failures by using people’s desire to protect their social connections (and social capital) and avoid any possible repercussions. Such repercussions could be economic and result in reduced trading partners for one’s business, and social and lead to loss of friends, or psychological and damage one’s self-esteem.

Most recently, La Ferrara (2003) studies kin groups in Ghana and finds that punishment in exacted not only on those who default, but also on the kin of those who default, and that the threat of such punishment induces compliance in the short run. These studies demonstrate that the relationship between social connections and group lending outcomes is complicated and worthy of further study. Dean S. Karlan (2007) find that both cultural similarity and geographic concentration lead to improved group lending outcomes (specially, higher repayment rates savings rates, and returns on savings). There is also suggestive evidence that social connections help groups distinguish between true negative shocks and mere reneging, and that those who have negative shocks are forgiven and thus allowed to continue borrowing.

3. The model
We assume that borrowers are fully aware of their own characteristics and other borrowers around them. But, lender does not know about this information domain of social network, in another word, lender does know who is risky and safe. The lender’s problem thus is that the borrowers share their private or hidden information, which is the relevant to the project. The lender would acquire the information, only the way to offer loan contract with borrowers.

The main mechanism we explore in this paper is joint liability. The lender could offer the contract to a group of borrowers. This would allow him to inter-link the borrower’s payoffs by making it contingent on her won as well as the payoff of her peer. The part of payoff that is contingent on her peer outcomes in the joint liability component of the payoff. The paper shows that this joint liability component is critical in discouraging the wrong kind of borrower and encouraging the right kind of borrower to borrow from the lender.

The model assume, a project requires an investment of 1 unit of capital and at the start at the period 1 and produces stochastic output $x$ at end of period 1. All borrowers have zero wealth and can thus only initiate the project if the lender agrees to lend to her. In a adverse selection model, the output depends on the type of the borrower undertaking the project. For simplicity, we assume that the project produces an output with positive value when it succeeds and zero when it fails.

We assume, a project undertaken by the borrower of type $i$ produce an output $x_i$ when it succeeds and 0 when it fails. The project will be succeeds with probability $p_i$ and fails with probability $1-p_i$. There are two type of borrower, the safe and risky type. The projects that risky and safe type’ undertake succeed with probability $p_r$ and $p_s$ respectively with $p_r<p_s$. That is, the risky type succeeds less often than the safe type. The probability of risky type and safe type is $\theta$ and $1-\theta$ respectively in the population. Then the expected payoff of an agent of type $i$ will be

$$U_i (r) = p_i (x-r)$$
Given that interest is paid only when the agent succeeds, the safe borrower’s utility is more interest sensitive as compared to the risky borrower’s utility since she succeeds more often.\(^1\)

Both types are impoverished with no wealth and have a reservation wage of \(\bar{u}\).

Suppose lender’s opportunity cost of capital is \(\rho\). We assume that the lender is operating in a competitive loan market and thus can no more than zero profit. This implies that the lender lends to the borrowers at a risk adjusted interest rate. The lender’s zero profit condition \(\rho = p_i r\) ensure that on a loan that has a repayment rate of \(p_i\), the interest rate charged is always

\[
ri = \frac{\rho}{p_i}
\]

It is important to note that competition amongst the lenders ensures that a particular lender can only choose whether or not to enter the market. He is not able to choose interest rate at which he lends. He always has to lend at the risk adjusted interest rate, at which he makes zero profit.

If borrower is risky type, then repayment will be \(p_r\) and if borrower is safe type then repayment will be \(p_s\). If lender does not know who is safe and who is risky, then average repayment is \(\bar{p} = \theta p_r + (1-\theta)p_s\). If the lender is not able to distinguish between safe and risky types, then only way in which he can discriminate between the two types is by inducing them to self select and reveal their hidden information.

In a pooling equilibrium, both type of borrower accept the same loan contract while in case of separating equilibrium, a particular loan is accepted by only one type. Socially viable project are those at which the output exceeds the opportunity cost of labour and opportunity cost of capital.

\[
pxi \geq \rho + \bar{u} \quad i = r, s; (2)
\]

That is the expected output of the project exceeds the wage rate of the borrower and the opportunity cost of capital invested in the projects.

### 3.1. Individual Lending

In perfect information world, the lender can identify the type of lending and choose to the contract accordingly. He would lend safe type at the interest rate \(r_s = \rho/p_s\) and to the risky type at the interest rate \(r_r = \rho/p_r\) given that \(pr < ps\).

The utility of a borrower type \(i\) that obtain the loan at her type specific interest rate \(r_i\) is given by following expression.

\[
U_i(r_i) = pxi - piri \geq \bar{u}
\]

This can be arranged to show that all social projects get financed with perfect information.

\[
pxi \geq \rho + \bar{u}
\]

This represents the first best equilibrium while perfect information is acquired by lender about the borrower characteristics.

In absence of the ability to discriminate between the risky and safe type borrower, the lender has no option but to offer a single contract. This contract may allow both type borrower (pooling equilibrium) or just offer one of the two types (separating equilibrium).

**Zero profit condition**- supposes that loan market is competitive and consequently profits are driven down to zero. The interest rate on the loan contract is determined by this zero profit condition in the loan market.

\[
ri = \frac{\rho}{p_i}
\]

There are three types interest rate possible in this condition-

\[
r_r = \frac{\rho}{p_r}
\]

\[\text{\(1\) As we see in the section on group lending, this leads to the safe type’s utility having a steeper slope than the risky types in the figure3.}\]
\[ r = \rho/p \]

\[ \bar{\rho} = \rho/\rho \]

\( r \) is the interest rate charged if the lender is convinced that the only safe borrower would take up the contract. Similarly, the lender would charge \( r \) if he think that only the risky borrower would take part in loan contract. If both type of borrower would self select in to the contract, the lender would charge \( \bar{\rho} \) where \( \bar{\rho} = \theta p_r + (1-\theta) p_s \). Since \( p_r < \bar{\rho} < p_s \) it follows that

\[ r_s < \bar{\rho} < r_r \]  \hspace{1cm} (3)

The lender would not like charge any other interest rate in the economy. Thus, participation in the credit market any type of borrower’s depend on these three types interest rate. There is a choice for lender to offer contract to safe type, risky type or both types of borrower. If the borrowing pool has both types, the lender’s pooling equilibrium is given by

\[ \bar{\rho} = \theta p_r + (1-\theta) p_s \]  \hspace{1cm} (4)

![Figure 1: Perfect Information Benchmark](image)

In this case, the interest rate would be \( \bar{\rho} = \rho/\rho \). The lender’s contract space is \([r_s, r_r]\) given that \( r_s < \bar{\rho} < r_r \).

**The constraints:** The lender has made sure that any contract that he offers satisfies the following condition.
Proposition 1: if the lender provides the borrower sufficient incentive to accept the loan contract, this condition should be satisfied.

\[ U_i (r, \ldots) \geq u. \]

Proposition 2: if each borrower type has the incentive to take the contract meant for her and does not have any incentive to try to win the other type, in separating equilibrium, the incentive compatibility condition should be satisfied.

\[ U_r(r, \ldots) > U_s(r, \ldots) \]
\[ U_s(r, \ldots) > U_r(r, \ldots) \]

Proposition 3: in a break even condition, lender’s profit should not be zero or cannot be more than zero. This is zero profit condition. Thus, in this case the lender’s break even condition and zero profit condition give us a condition that bind with equality.

\[ r_i = \rho / p_i \]

3.2. Group Lending

For explanation of group lending and joint liability, paper explores the simplified version of Ghatak Model (1999, 2000). The contract that the lender offers the group is such that the members within the group are jointly liable for each other’s outcome. If a borrower succeeds, she pays the specified interest rate (r). Further, if her peer fails; she is required to pay an additional joint liability component (c). The lender offers a joint liability contract (r, c). If borrower’s project fails, the limited liability constraint applies and borrower does not pay anything. A borrower’s payoff in the group lending is given by:

\[ U_i (r, c) = p_i p_j (x_i - r) + p_i (1 - p_j) (x_i - r - c) \]
\[ = p_i (x_i - r) - p_i (1 - p_j) c \] (5)

With probability \( p_i \), the borrower succeeds. If she succeeds, repay r and her payoff would be \( (x_i - r) \). With probability \( p_i (1 - p_j) \), she succeeds but her peer fails. In this case she has to make the joint liability payment c. Given the contract environment; lender requires that the borrower self-select into groups before apply for a loan.

3.2.1. Matching problem

In contract lending theory, matching is two types, one is Positive Assortative Matching and other is Negative Assortative Matching. With positive assortative matching, the groups would either have safe types or both risky types. In case of negative assortative matching, each group would have both safe and risky types.

Proposition 4 (Positive Assortative Matching): Joint liability contract described above represent positive assortative matching.

It is important to see deeply what happen in case of positive assortative matching due to the joint liability c. Due to joint liability payment, everyone want safe partner (Armendariz de Aghion, and Jonathan M., 2005). The safer the partner, the lower the probability of incurring the joint

\(^{2}\) Interest rate includes principal plus rate of interest.

\(^{3}\) The additional joint liability payment which is incurred if the borrower succeeds but her peer fails.
liability payment $c$ due to her peer failure. Here we examine the benefits accruing to the risky type by taking on a safe peer and the loss incurred by the safe type by taking on a risky peer.

$$U_{rs}(r, c) - U_{rr}(r, c) = p_r (p_s - p_r) c \quad (6)$$
$$U_{ss}(r, c) - U_{sr}(r, c) = p_s (p_s - p_r) c \quad (7)$$
$$P_s (p_s - p_r) c > p_r (p_s - p_r) c \quad (8)$$

Equation (6) tells us the gain accruing to the risky type from pairing up with safe type instead of risky type. Equation (7) gives us the loss incurred by a safe type from pairing up with a risky type instead of another safe type and equation (8) compares the two equations that (7) is smaller than (8). It follows that

$$U_{ss}(r, c) - U_{sr}(r, c) > U_{rs}(r, c) - U_{rr}(r, c) \quad (9)$$

This explores the idea that the safe type’s loss is more than risky type’s gain. So, risky type would not be able to agree to safe type because is safe type offer to risky type, compensation would be more if risky type borrower fails. So, joint liability contract leads to only positive assortative matching where by a safe type pairs up with another safe type and risky type pairs with another risky type.

**Proposition 5** (Socially Optimal Matching): positive assortative matching maximizes the aggregate expected payoff of all borrowers over different matches.

Rearranging equation (8) for proof of this statement and we find-

$$U_{ss}(r, c) + U_{rr}(r, c) > U_{rs}(r, c) + U_{sr}(r, c) \quad (10)$$

This implies that positive assortative matching maximizes the aggregate expected payoff of all borrowers over different matches.

### 3.2.2. Indifference curves

From equation (5), suppose that-

$$U_{ij}(r, c) = p_i (x_i - r) - p_i (1 - p_j) c = \bar{k}$$

Differentiate $c$ with respect to $r$, we can find-

$$\left. \frac{dc}{dr} \right|_{U_{ii}} = \text{constant}$$

$$= - \frac{1}{1 - p_i}$$

This implies that the safe type’s indifference curve is steeper than the risky type’s indifference curves.

$$\left| \begin{array}{c} \frac{1}{1 - p_s} \\ \frac{1}{1 - p_r} \end{array} \right| > \left| \begin{array}{c} \frac{1}{1 - p_s} \\ \frac{1}{1 - p_r} \end{array} \right|$$

This comparatively represent that the safe type is less concerned about the joint liability payment $c$ because she is paired up with a safe type. She would like to get a low interest rate $r$ and would
happily trade off a higher joint liability payment \( c \) in exchange. While, we can see that it is rational for the risky type to dislikes the joint liability payment comparatively more. So, risky type’s made pair with risky type and would prefer to have a lower joint liability payment down and does not mind the resulting increases in interest rate. The lender can use this result the fact that the safe groups and the risky groups trade off the joint liability payment and interest rate payment at different rates. Figure 2 shows this result.

![Indifference Curves](image)

**Figure 2: Risky and Safe Types’ Indifference Curves**

Now, we are coming lenders’ problem. There are two instrument in the contract \((r, c)\), which lender can use in contract so that two types borrower trade off \( r \) with \( c \) at different rate and induce them to self select into contract. The lender offers contracts \((r_r, c_r)\) and \((r_s, c_s)\) and designs the contracts in such a way that the risky borrower choose first contract and safe borrower choose second contract. Such contract maximizes the borrower’s payoff subject to the following constraint:

\[
 r_r p_r + c_r (1- p_r) p_r \geq \rho \quad \text{and} \quad \frac{dc}{dr} = - \frac{1}{1- p_r} \quad (11)
\]

*Equation (11) represents the lender’s zero profit condition for risky type’s borrower.*

\[
 r_s p_s + c_s (1- p_s) p_s \geq \rho \quad \text{and} \quad \frac{dc}{dr} = - \frac{1}{1- p_s} \quad (12)
\]

*Equation (12) represents the lender’s zero profit condition for safe type borrower.*

\[
 U_{ii} (r_i, c_i) \geq u_i, \quad i = r, s \quad (13)
\]

*Equation (13) represents the participation constraint for i type borrower.*

\[
 x_i \geq r_i + c_i, \quad i = r, s \quad (14)
\]

*Equation (14) represents the limited liability constraint for type i borrower.*
\[ U_{rr} (r, c_r) \geq U_{ss} (r_s, c_s) \]  
\[ U_{ss} (r_s, c_s) \geq U_{ss} (r, c_r) \]  
Equation (15) and (16) represent the incentive compatibility constraint for group \((i, i)\).

For optimal contract that allows the lender to distinguish between safe and risky type borrower, we need to define the optimal point \((r^*, c^*)\). This is the point where lender’s zero profit condition curves for safe type borrower and lender’s zero profit condition curve for risky type borrower is intersect to each other.

\[ r^* = \frac{(p_s + p_r - 1)}{p_s p_r} \rho \] and
\[ c^* = \frac{1}{p_s p_r} \rho \]

Optimal zero profit condition is shown in figure 3.

**Proposition 6** (separating equilibrium) for any joint liability contract \((r, c)\)

- If \(r_s < r^*\), \(c_s > c^*\), then \(U_{ss} (r_s, c_s) > U_{rr} (r_s, c_s)\)
- If \(r_r > r^*\), \(c_r < c^*\), then \(U_{rr} (r_r, c_r) > U_{ss} (r_r, c_r)\)

The safe groups prefer joint liability payment higher than \(c^*\) and interest rates lower than \(r^*\) while risky groups prefer joint liability payment lower than \(c^*\) and interest rates higher than \(r^*\). With joint liability payment, the lender is able to charge each type borrower on different interest rate. This allows lender to charged different interest rate on different type of borrower.
3.2.3. Optimal Contracts
In joint liability theory, there are potentially two types of optimal contract. The first kind is separating contract. The separating contracts are where the safe group’s contract is north-east on the safe type’s indifference curve and risky group’s contract which is south-west on the risky type’s indifference curve. The second kind of contract is the pooling contract at \((c^*, r^*)\).

3.3. Self Financing and their Effect on Borrower’s Payoffs, Penalty size and Interest Rate
Above discussion is based on assumption that entire funding of the project has been done by the funding agencies. Now, we assume that some part of the project’s requirement is complemented by borrowers own saving and also assume that the self financing (borrower’s own savings) is too costly. In this paper, we want to see the effect of self financing on borrower’s payoffs, size of penalty which is imposed by the lending institution and interest rate.

Now, suppose that a borrower can signal the quality of his business project to a lending institution by self financing a fraction of \(\Psi\) of this project. Further, suppose that the opportunity cost of funds to a borrower is \(\lambda^4\). In this setting, we now want to describe the borrower’s payoff as a function of the type of his business project, the loan repayment \(r\), and the self financing fraction \(\Psi\). If a borrower self finance a fraction \(\Psi\) of his business project and repay \(r\) to the lending institution for the rest of this project then his expected payoff for a business project can be calculated from the equation (5)

\[
U_{ij}(r,c) = p_i [x_i - (1-\Psi) r] - p_i (1-p_j)c - \Psi (1+\lambda) \tag{17}
\]

If we compare this equation with equation (5), we can get that project’s payoff with saving is higher than without saving. The reason is that first section of right side of equation (17) indicates as the value of self financing increases, the value of \((1-\Psi)\) will be smaller and \([x_i - (1-\Psi) r]\) will be higher. Third section of this equation tells us that self financing and opportunity cost of the saving should be deducted from the payoff. Subtracting equation (5) from equation (17), we can obtain-

\[
U_{ij}(r,c) - U_{ij}(r,c) = \Psi(p_i + 1 + \lambda) \tag{18}
\]

Finally we conclude that payoff with self financing is greater than the payoff without self financing.

There is the possibility that size of penalty which is imposed by lending institutions on borrowers, should be reduce because savings of the group is deposited in bank in the name of the group specially in the case of SHG-Bank Linkage Model in India and ROSCA Model in the rest of the world. If borrower going to default, the bank can deduct the unpaid amount of loan from the SHG Bank account. So, here, peer pressure may be possible tool by which other member of the group put pressure on defaulting member to repay her share. Thus, the possibility is that the members do less try to going default if the profit from default is less than the loss from default.

But, in the case of MFIs, where MFI have not any control on borrowers saving, it would be difficult to predict that size of penalty will be reduced. Thus self financing would be an incentive for the borrower that she does hard work for success of her project.

\(^4\) Note: \(\lambda\) should be less than the opportunity cost of the fund, if this is not happen, then the borrowers would be

incentive to use money another way and gain profit.
On interest rate front, group savings provide liquidity pool to the bank, and there is less possibility of default and in same time, bank get a security in form of group savings. So, risk on lending is less compare to lending without group saving. There is the possibility that bank will reduced interest rate and spread their credit services. It is less optimal to MFIs. But, still it is profitable to the MFIs to reduced interest rate and spread their financial services, if self financing is happened. Self financing will act like a security for both borrowers and lenders, because it will create pressure on borrower to do hard work for success of her project and success project leads to high repayment rate.

4. Policy Issues
In that world where rural credit market is imperfectly competitive and acquiring information about borrower’s type is not costless, joint liability mechanism of group lending is best option to solve these types of market imperfections. In joint liability model, lenders have two policies variable \((r, c)\) by which he try to do make positive assortative matching and reduce risk of lending.

In recent environment, basically in case of Indian microfinance, where borrower’s suicide emerges as critical issues in policy arena, self financing policy may be hope for whole industry. Government should try to make an environment where some portion of project funding done through self financing. So that risk of the market can be reduced.

5. Conclusion
The paper successfully shows that joint liability contract lead to positive assortative matching within groups. Once the matching process takes place, the lender is able to distinguish between who is safe and who is a risky group by using the contract variables \(r\) and \(c\). we try to show that if risky type pair with safe type the gain from this pairing will be smaller than the loss, if safe type pairing with risky type. The reason behind this argument is that the penalty imposed by lender is high for risky borrower and low for safe borrower because the probability that project will be succeeds is high for safe borrower and low for risky borrower.

Another finding is that positive assortative matching maximizes the aggregate expected payoffs of borrowers over all possible matches.

\[
U_{ss}(r, c) + U_{rs}(r, c) > U_{sr}(r, c) + U_{sr}(r, c)
\]

First part of this expression present positive assortative matching and second part presents negative assortative matching. We can see clearly that if risky borrower is going to default then \(U_{rr}(r,c)\) will be zero and gain from \(U_{rs}(r,c)\) is less than loss from \(U_{sr}(r,c)\), so still first part of expression is higher than the second part because gain from \(U_{ss}(r,c)\) is high than gain from second part. So, positive assortative matching is socially optimal.

The third conclusion is that the interest rate is lower in joint liability lending than individual lending. Figure 3 shows that in socially optimal joint liability matching, interest rate will be \(r^*\) because lender do solve the informational problem about borrowers by joint liability and try to control of default with contract variable \(r\) and \(c\). But in case of individual lending without collateral, lender does not have any information about borrower that who is risky and who is safe. So, finally the interest rate become high than the joint liability interest rate \(r^*\).

The final conclusion is that the self financing may be important tool for the spread of micro credit services. Self financing can bring down the interest rate and penalty size and improve the expected payoff of the borrowers. But, there is need to more empirical test on this issue.
References: