ROLE OF THE IMF UNDER SIGNAL NOISE

Jun-Hyung Ko
Hitotsubashi University

Abstract
This paper theoretically analyzes the early warning system (EWS) of the IMF based on the principal-agent model. We search for the trade-off of the optimal contract of the IMF under interim intervention and signal noise. The main findings are as follows. First, when the net loss coming from noise under a good fundamental is higher than the net gain by an interim intervention under a bad fundamental, the debtor country exerts less effort as the noise effect becomes larger. Second, when the net loss in a good fundamental is smaller than the net gain in a bad fundamental, an accurate signal may give rise to the moral hazard problem. Third, when the marginal utility by the intervention of the IMF is higher on bad fundamentals than on good fundamentals, the higher ability of the IMF to mitigate the crisis will elicit a weak policy effort from the country. On the other hand, when the economy has higher marginal utility in case of good fundamentals, deeper intervention of the IMF offers an incentive of a stronger policy effort to the country. Fourth, mandating the IMF to care about a country's welfare as well as safeguarding its resources does not necessarily imply that the debtor country will exert less effort.

Keywords: IMF, EWS, principal-agent model, optimal contract

JEL classification: F34; F21

1 INTRODUCTION
As more developing countries liberalize their capital control regulations and as more investors invest huge amounts of money abroad, the possibility of a financial crisis is increasing. Financially vulnerable countries are always at the risk of a currency crisis in exchange with chance of welcoming beneficial capital flows. The IMF is expected to take necessary actions to prevent a crisis by forecasting and advising a developing country's authorities. The EWS seems to be a good tool for this challenging work. The IMF is expected to help developing countries build a necessary and reliable economic statistical database. It is a foundation of every EWS studies for crisis prevention. The prevention of a possible crisis is related to both the effort of both the program country and the IMF.

This paper focuses on two roles of the IMF to ensure the stability of the debtor country's...
fundamentals and the financial market: its role as the international lender of last resort (ILOLR) and its role in the countries' surveillance with regard to the application of EWS. While its role as an ILOLR is a way of providing short-term liquidity to crisis countries subject to appropriate conditions, its ``surveillance'' role involves effective monitoring that would limit the extent of debtor moral hazard, and redress the problem through policy consultation. The EWS meant to predict a currency crisis, to warn the debtor country in advance and, therefore, to prevent a severe crisis. However, if the action of the IMF in the interim period may lead to another exposure to international liquidity cycles, it may cause an unnecessary crisis.

The recent literature focuses on the implication of asymmetric and private information for the behavior of debtor countries and speculators. Botmand and Diks (2005) analyze the speculative attacks based on the global games developed by Morris and Shin (1998, 2002). Chami, Sharma, and Shim (2004) find the role of the coinsurance arrangement among debtor countries to safeguard themselves against the currency attacks based on the principal-agent theory. These theories divide the time period only into two parts: before the crisis and after the crisis. There are many empirical papers that investigate the effect of EWS but there is none use a theoretical approach.

In this paper, I develop a simple principal-agent model with three parts: before the crisis, the interim period, and the final period. This paper focuses on the interim intervention of the IMF under the EWS approach. We explore the incentive effects of the IMF's financing and analyze the trade-off when the anticipation of the IMF is suffering from noise. There is a benefit when the IMF warns earlier and the fundamental is in a bad situation. In particular, we analyze two cases: the case where the IMF warns in advance although the fundamental is good and the case where the IMF does not warn although the fundamental is bad. We also analyze the optimal level of the contract considering the IMF as an ILOLR.

The main results are as follows. First, when the net loss of an inaccurate signal under a good fundamental is higher than the net gain by proper intervention under a bad fundamental, the debtor country exerts less effort as the noise effect becomes larger. A signal that frequently gives false alarms will definitely make the world economy confused or have countries skeptical about the warning system, and as a result, they will become less willing to take necessary action. Therefore, the IMF should minimize the noise so that the country becomes more willing to take major policy actions. Second, when the net loss in a good fundamental is smaller than the net gain in a bad fundamental, an accurate signal may give rise to the moral hazard problem. Intentional concealing of the information may have positive effects to solve this problem. Third, when the marginal utility from the intervention of the IMF is higher on bad fundamentals than on good fundamentals, the stronger ability of the IMF to mitigate the

\[ \text{2) For example, see Abdul (2002) and Ito and Orii (2009).} \]
A crisis will elicit a weaker policy effort from the country. On the other hand, when the economy has higher marginal utility in the case of good fundamentals, the deeper intervention of the IMF offers an incentive of a stronger policy effort to the country: although the noise lets the economy face the shock, the higher intervention mitigates the shock, and hence, the country is induced to deliver a stronger policy response. Fourth, mandating the IMF to care about a country’s welfare as well as safeguarding its resources does not necessarily mean that the debtor country will exert less effort.

The rest of the paper is organized as follows. Section 2 presents a model to explain how the IMF influences the macro-policy of debtor countries using the framework of a principal-agent model. Section 3 examines the effectiveness of the IMF’s lending based on the characteristics of the IMF. Section 4 concludes.

2 THEORETICAL APPROACH

In this section, based on the principal-agent model, we examine the incentive effects of the IMF’s intervention and the trade-offs under some signal noise. We try to assess how public intervention can affect the scale of capital flows and the welfare of a country.

2.1 Benchmark case 1: ex post intervention

The principal is the IMF and the agent is the debtor country. We assume that the country has a von Neumann-Morgenstern utility function:

\[ U(y, e) = u(y) - v(e) \]  

where \( u \) is a continuously differentiable concave function with \( u' > 0 \) and \( u'' < 0 \), \( y \) is the net output, and \( e \) is the effort level of the debtor country. Policy efforts result in cost, \( v(e) \), where \( v \) is a convex function: \( v' > 0 \) and \( v'' > 0 \). The output is given by

\[ y = \lambda L - rL \]  

where \( L \) denotes the international capital flows into the domestic country, \( \lambda L \) is a production function, and \( \lambda \) is the productivity of the debtor country\(^3\), and \( r \) is the return rate promised to the international investors.

We assume that the country is subject to a crisis probability \( \theta \) and the exogenous shock leads to a loss of output. The expected utility of the debtor country is

\[ E(U) = (1 - \theta(e))u(L_G - rL) + \theta(e)u\left(\frac{1 - \frac{\alpha^2}{\sigma^2_F}}{\lambda^2}L_B - \beta rL\right) - v(e) \]  

where \( L_G \equiv \lambda_G L \) is the output under a good fundamental while \( L_B \equiv \lambda_B L \) is the output

\(^3\) It is possible to assume an increasing-return-to-scale production function, for example, \( L^\lambda \) with \( \lambda > 1 \). Then, the volume of capital inflow is dependent on the productivity of the debtor country.
under a bad fundamental. We assume that \( \lambda_G > \lambda_B \): productivity is higher under a good fundamental. Further, here \( \alpha_3 \) is the damage level that lowers the output of the debtor country in the final period, \( \sigma_f \) is the parameter that reflects the efficacy of the IMF to reduce the output losses in the final period, and \( \beta \) is the discount factor of the international investors and reflects the loss from drawing capital back in the interim period. The first term in the right-hand side is the expected return of the debtor country with a good fundamental and the second term is the expected return when the economy is facing a crisis. We also assume that the policy effort \( e \) can reduce the cost: \( \theta' < 0 \) and \( \theta'' > 0 \).

The IMF has a fixed amount of resources, \( x \), as an endowment, and it can intervene and make contingent loans to the crisis-facing debtor country. It is assumed that the IMF is also concerned about the debtor country's utility. Thus, the utility function becomes

\[
U_{\text{IMF}}(x - \sigma, y) = \tilde{u}(x) + \gamma u(y),
\]

where \( \tilde{u}(x) \) implies that the IMF's utility positively depends on the size of its own resources, \( \sigma \in \{\sigma_1, \sigma_f\} \) is defined as the cost of the IMF to diminish the bad effect of the crisis, \( u(y) \) is the debtor country's utility function, and \( \gamma (\in [0,1]) \) is the relative weight of the IMF's direct concern for the welfare of the debtor country.

### 2.2 Case 2: interim intervention with perfect signal

In case 2, we assume that the IMF can observe the fundamental of the debtor country beforehand. Here, too, there are two situations: high output with a good fundamental and interim intervention under a bad fundamental. In this case, the IMF can set a proper intervention level so that it can guide the debtor country to keep its fundamental good. The expected utility of the debtor country becomes

\[
E(U) = (1 - \theta(e))u(L_G - rL) + \theta(e)u \left( \frac{1 - \alpha_2}{\sigma_f} \right) L_B - \beta rL - v(e),
\]

where \( \sigma_f \) is a parameter that reflects the efficacy of the IMF to reduce the output losses in the interim period and \( \alpha_2 \) is the crisis level in the interim period. The first term is the expected return of the debtor country under a good fundamental. The second term is the expected return when the IMF intervenes and mitigates the crisis under a bad fundamental country. Case 2 is assumed to be always better than benchmark case 1. In the next subsection, we consider the case where there exists noise in the prediction of the IMF. We examine how the economy and the IMF suffer from noise and how noise distorts the economy.

### 2.3 Case 3: interim intervention under noise

We assume that the timing of the principal-agent process is as follows.

1) The role of the IMF and its ability to mitigate the crisis (\( \sigma \)) is precommited. The
probability of sending a wrong message ($\varepsilon$) is common knowledge.

2) International capital ($L$) flows as long as the expected return from domestic investment is higher than that on the world markets. We assume that there is an elastic supply of international investors willing to enter the domestic market.

3) After receiving $L$, the debtor country chooses the level of effort ($e$) to foster the higher net expected output and not to fall into a crisis.

4) Nature roles the dice ($\theta(e)$) and the market reaction is then recorded. The probability function $\theta(e)$ is a decreasing convex function of $e$. Thus, given $\varepsilon$, four output types are realized.

5) The EWS of the IMF expects output realization $L_H$ or $L_L$ with probability $\varepsilon$.

6) If the country faces a crisis and requests help ($\sigma$) from the IMF, the IMF manages the crisis.

![Game Tree](image)

**Figure 1** Game Tree

### 2.4 EWS

The IMF has a role in the interim period. It has access to an imperfect signal on the state of
the debtor country's finances in the interim period. The IMF has a signal to indicate whether the country is making enough efforts and has sufficient resources to pay the debt in full. Based on this information, the IMF pronounces its view of the current state of the fundamental and chooses whether or not to intervene. The joint distribution over the signals and the real state of fundamentals is given in Table 1.

<table>
<thead>
<tr>
<th>IMF Signal</th>
<th>Good</th>
<th>Bad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamentals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bad</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: 1) The rows give the conditions of the fundamental and the columns provide the messages from the IMF. 2) $\theta$ is the probability that the country enters a crisis. 3) $\varepsilon$ is the degree of noise in the IMF's signal.

**Table 1** IMF as a whistle blower (EWS)

The IMF's signal is imperfect in two ways. First, the signal space is binary, \{Good, Bad\}, and only tells whether the fundamental is good or bad. Thus with this simple signal system, it may be hard to tell how bad the fundamental is. Second, the binary signal suffers from noise. Even when the signal is incorrect, the market reacts.

- **[Fundamental, IMF signal] = [Bad, Bad]** (Relevant intervention with signal): In this case, the anticipation is correct and the economy enters a bad phase. The IMF takes a measure so that the economy recovers soon under the IMF’s discipline. The IMF attenuates the effect of the parameter $\alpha_2$ by a factor $\sigma_I$, which stands for the efficiency of intervention in the interim period, and the crisis is overcome.

- **[Fundamental, IMF signal] = [Good, Bad]** (Irrelevant intervention with noise): In the interim period, the debtor country enters a good phase and has enough resources, but the IMF provides the wrong signal. Therefore, due to the coordination game among the international investors, the wrong signal results in the economy decreasing by as much as $\alpha_1$, and the IMF enters to mitigate the self-fulfilling shock. The output is less than that under the correct signal. Therefore, in this model, the intervention efficiency of the IMF in the interim period decreases as the noise increases.

- **[Fundamental, IMF signal] = [Bad, Good]** (Non-intervention with noise): When the IMF fails to detect the actual situation of the country's fundamental, its intervention is delayed. Therefore, the damage ($\alpha_3$) is the maximum and the country is exposed to the full impact of the crisis in the final period. As compared to the two former cases, the intervention of the IMF occurs later after the debtor country is fully engulfed in the crisis. Consequently, although the IMF fails to provide the correct signal early, it
lessens the crisis by $\sigma_F$ in the final period. It is assumed that accurate intervention in the interim period is more efficient than that in the final period.

2.5 Debtor country’s objective function

The expected utility of the debtor country becomes

$$E(U) = (1 - \theta)(1 - \varepsilon)u(L_G - rL) + (1 - \theta)\varepsilon u\left[1 - \frac{\alpha_2}{\sigma_i}\right]L_G - \beta rL$$

$$+ \theta\varepsilon u\left[1 - \frac{\alpha_3}{\sigma_F}\right]L_B - \beta rL + \theta(1 - \varepsilon)\left[1 - \frac{\alpha_3}{\sigma_i}\right]L_B - \beta rL - v(\varepsilon),$$

where $\sigma$ is the parameter that reflects the efficacy of the IMF to reduce the output losses.

The first term is the expected return of the debtor country with a good fundamental and an accurate signal. The second term is the expected return with a good fundamental and an inaccurate signal. The third term is the expected return with a bad fundamental and an inaccurate signal. Finally, the fourth term is the expected return with a bad fundamental and an accurate signal. Henceforth, we consider the case where $\varepsilon$ is not zero. Table 2 provides the final output realization based on the four output types.

<table>
<thead>
<tr>
<th>(fundamental, IMF)</th>
<th>Output Realization</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Good, Good</td>
<td>$L^\lambda - rL$</td>
<td>$(1 - \theta)(1 - \varepsilon)$</td>
</tr>
<tr>
<td>b) Bad, Good</td>
<td>$\left(1 - \frac{\alpha_3}{\delta}\right)Y - \beta rL$</td>
<td>$\theta\varepsilon$</td>
</tr>
<tr>
<td>c) Good, Bad</td>
<td>$\left(1 - \frac{\alpha_3}{\delta}\right)Y - \beta rL$</td>
<td>$(1 - \theta)\varepsilon$</td>
</tr>
<tr>
<td>d) Bad, Bad</td>
<td>$\left(1 - \frac{\alpha_3}{\delta}\right)Y - \beta rL$</td>
<td>$\theta(1 - \varepsilon)$</td>
</tr>
</tbody>
</table>

Note: The second column shows the following: a) the income level when the fundamental is good and the signal is good, b) the income level when the fundamental is good and the signal is bad, c) the income level when the fundamental is bad and the signal is good, and d) the income level when the fundamental is bad and the signal is bad.

Footnote 4: For simplicity, the following assumptions are made without loss of generality.

Assumption 1. $\alpha_3 > \alpha_2 > \alpha_1 > 0$.

The case with a bad fundamental and a good signal is the worst case with the lowest outcome.

Assumption 2. $1 > \frac{\alpha_i}{\sigma}, \quad i \in \{1,2,3\}$: the IMF has the ability to ameliorate the crisis.

Assumption 3. The output under the IMF program is higher than that when not taking assistance from the IMF; further, this is a sufficient condition for the debtor country to seek help from the IMF.
income level when the fundamental is bad and the signal is bad.

**Table 2 IMF as a firefighter (as an ILOLR)**

Assuming the interior solution, \( e \) satisfies the following first-order condition:

\[
\frac{\partial E(U)}{\partial e} = \theta'(e) \left[ \alpha u \left[ \left( 1 - \frac{\alpha_i}{\sigma_i} \right) L_b - \beta r L \right] + (1 - \varepsilon) u \left[ \left( 1 - \frac{\alpha_i}{\sigma_i} \right) L_b - \beta r L \right] \right] - \nu'(e) = 0.
\]

This equation implies that the policy effort is chosen such that the marginal utility equals the marginal cost of the policy to overcome the policy. Since \( e^* > 0 \), \( \theta'(e) < 0 \) and \( \nu'(e) > 0 \), we have

\[
(1 - \varepsilon) u(L_G - rL) + \alpha u \left[ \left( 1 - \frac{\alpha_i}{\sigma_i} \right) L_G - \beta r L \right] > \alpha u \left[ \left( 1 - \frac{\alpha_i}{\sigma_i} \right) L_G - \beta r L \right] + (1 - \varepsilon) u \left[ \left( 1 - \frac{\alpha_i}{\sigma_i} \right) L_G - \beta r L \right],
\]

which implies that the expected return on a good fundamental is higher than that on a bad fundamental even though noise exists. From (7), we can obtain the effect of the variables \( \{e, \sigma_i, \sigma_F\} \) on the policy effort \( e^* \).

**Definition 1:** “The net benefit from the IMF’s help” is the positive gap between the two results given a bad fundamental, \( u \left[ \left( 1 - \frac{\alpha_i}{\sigma_i} \right) L_B - \beta r L \right] - u \left[ \left( 1 - \frac{\alpha_i}{\sigma_i} \right) L_B - \beta r L \right] \), to ameliorate the crisis under a bad fundamental.

**Definition 2:** “The net loss in the strong effort exerted due to the noise” is the positive gap between the two results given a good fundamental, \( u(L_G - rL) - u \left[ \left( 1 - \frac{\alpha_i}{\sigma_i} \right) L_G - \beta r L \right] \), when the country has a good fundamental.

The net benefit is obtained under the situation where the IMF can help the debtor country earlier given a bad fundamental. This is the purpose of the EWS. However, the net loss incurred under the situation where the signal is wrong given a good fundamental. Therefore, the signal is another factor to determine the effort level of the debtor country.

Using the implicit function theorem, we can derive the effect of the signal and its noise on the policy effort \( e \).

**Proposition 1:** When the net loss in the strong effort exerted due to the noise is higher than the net benefit from the IMF’s help, the smaller the signal noise, the larger is the country’s
effort: $\frac{\partial e}{\partial \theta} < 0$; When the net benefit is bigger than the net loss in the strong effort exerted, the larger the signal noise, the larger is the country’s effort: $\frac{\partial e}{\partial \sigma} > 0$.

The first case of Proposition 1 refers to the scenario where the net loss under a good fundamental is higher than the net gain under a bad fundamental. In such a case, the debtor country exerts less effort as the noise effect becomes larger. Here, it is possible that the utility under a bad fundamental and a bad signal is not that different from that under a bad fundamental and a good signal. In this case, the low effort level is not affected by the noise although the economy enters a bad phase. However, when the utility under a good fundamental and a bad signal deviates far from that under a good fundamental and a good signal, the high effort level is quite affected by the noise. When the noise level is large, the debtor country does not want to make any effort. However, as the noise level decreases, the country does not have to care about the failure of the IMF’s signal when taking major policy actions. The second case of Proposition 1 refers to the scenario where the benefit from the IMF’s interim intervention is larger under a bad fundamental. This means that the EWS can effectively minimize the crisis. However, Proposition 1 implies that the damage from the crisis to the debtor country is minimized with little effort, and therefore, leads to the moral hazard problem. Ironically, it is only when the noise level becomes high that the debtor country make an effort.

**Corollary 1**: When the net loss is higher than the net benefit from the IMF’s help, the higher the noise level, the higher is the probability of the crisis occurring: $\frac{\partial \theta}{\partial e} < 0$.

**Corollary 2**: When the net benefit under the noise is bigger than the net loss, the higher the noise level, the lower is the probability of the crisis occurring: $\frac{\partial \theta}{\partial e} > 0$.

Corollary 2 yields a very interesting result. If the IMF can control the public information, it is optimal not to release all information to the market. The debtor country makes every effort not to fall into the worst situation. This is a possible solution to prevent the moral hazard problem. Propositions 2 and 3 show the relationship between the help provided by the IMF in the interim and final periods and the effort made by the debtor country.

**Proposition 2**: The larger the value of $\theta$, the smaller is the policy effort: $\frac{\partial e}{\partial \sigma} > 0$.

**Proposition 3**: When the marginal utility of a country from the intervention is higher under a bad fundamental and a bad signal than that under a good fundamental and a wrong signal, $\frac{\partial e}{\partial \sigma} < 0$. When the marginal utility under a good fundamental and a wrong signal is higher, the effort

5) For the proof, see Appendix B.
level increases: \( \frac{\sigma}{\sigma_f} > 0 \).

Proposition 2 is very straightforward. If the IMF helps more in the worst case, the country exerts less effort. Proposition 3 compares the following two cases: a good fundamental and a wrong signal, and a bad fundamental and a correct signal. Proposition 3 implies that the effort level is dependent on whether or not a signal is correct. When the marginal utility under a bad fundamental and a correct signal is higher than that under a good fundamental and a wrong signal, the smaller the mitigation effect (the greater the intervention of the IMF), the more is the effort exerted by the country. Propositions 2 and 3 show when the marginal utility from the intervention of the IMF is higher under a bad fundamental than under a good fundamental, the higher ability of the IMF to mitigate the crisis will elicit a weaker policy effort \( e^* \) from the debtor country. On the other hand, when the economy has higher marginal utility under a good fundamental, a deeper intervention by the IMF offers an incentive of a stronger policy effort \( e^* \) to the country: although the noise results in a shock for the economy, the higher \( \sigma \) mitigates the shock, and hence, the country is induced to provide a stronger policy response.

3 OPTIMAL CONTRACT OF THE IMF

In this section, we find the optimal contract of the IMF. The expected utility function of the IMF is

\[
EU_{IMF} = (1 - \theta)(1 - \varepsilon)\tilde{u}(x, \varepsilon) + \gamma u_L G - rL \]

\[+ \left(1 - \theta\right)\varepsilon \left[ \tilde{u}(x - \sigma_f, \varepsilon) + \gamma u_L \left(1 - \frac{\alpha_f}{\sigma_f} \right) L_g - \beta r L \right] \]

\[+ \theta\varepsilon \left[ \tilde{u}(x - \sigma_f, \varepsilon) + \gamma u_L \left(1 - \frac{\alpha_f}{\sigma_f} \right) L_g - \beta r L \right] \]

\[+ \theta(1 - \varepsilon) \left[ \tilde{u}(x - \sigma_f, \varepsilon) + \gamma u_L \left(1 - \frac{\alpha_f}{\sigma_f} \right) L_g - \beta r L \right] \]

\[- \gamma v(\varepsilon). \tag{9}\]

The IMF’s problem of choosing the optimal loan contract can be specified as follows:

\[
\max_{\sigma_f, \varepsilon} EU_{IMF}(\sigma_f, \varepsilon; e^*) \tag{10}\]

s.t. \( e^* = \arg \max_{\varepsilon} EU(\sigma_f, \varepsilon) \)

\[EU(e^*; \sigma_f, \varepsilon) \geq \bar{U}. \tag{11}\]

Equation (10) is the incentive compatibility constraint for the debtor country and equation
(11) is the participation constraint for the debtor country. From assumption 3, the debtor country prefers to be a member of the IMF.

The principal-agent optimal contract can be solved by backward induction. First, we solve the expected utility maximization problem for the debtor country given \( \{\varepsilon, \sigma_j, \sigma_F\} \). The solution \( (\varepsilon^*) \) becomes the optimal value to satisfy the incentive compatibility constraint. Next, given the solution, the IMF offers the optimal contract or precommits its discipline \( \{\sigma_j, \sigma_F\} \).

### 3.1 IMF objective: balancing country welfare and safeguarding resources

The first-order conditions for the IMF’s utility maximization with respect to the contract variables \( \{\sigma_j, \sigma_F; \varepsilon\} \) are

\[
0 = -(1 - \theta) \left[ \frac{\partial u(x - \sigma_j, \varepsilon)}{\partial \sigma_j} \right] - \theta(1 - \varepsilon) \left[ \frac{\partial u(x - \sigma_j, \varepsilon)}{\partial \sigma_j} \right] - \frac{\partial \theta}{\partial \varepsilon} \left[ \frac{\partial (1 - \varepsilon)u(x, \varepsilon) - \varepsilon u(x - \sigma_F, \varepsilon) - (1 - 2\varepsilon)u(x - \sigma_j, \varepsilon)}{\partial \sigma_j} \right] \tag{12}
\]

\[
0 = -\frac{\partial u(x - \sigma_j, \varepsilon)}{\partial \sigma_j} - \gamma \left[ \frac{\partial u(x - \sigma_F, \varepsilon)}{\partial \sigma_j} \right] - \frac{\partial \theta}{\partial \varepsilon} \left[ \frac{\partial (1 - \varepsilon)u(x, \varepsilon) - \varepsilon u(x - \sigma_F, \varepsilon) - (1 - 2\varepsilon)u(x - \sigma_j, \varepsilon)}{\partial \sigma_F} \right] \tag{13}
\]

**Definition 3:** The expected utility of the debtor country when the IMF only considers safeguarding resources \((\gamma = 0)\) is defined as follows:

\[
EU^{\gamma=0} = E^{\gamma=0} = u\left(1 - \frac{\alpha_j}{\sigma^*_j} \right) L_g - \beta r L \right) + (1 - E^{\gamma=0}) u\left(1 - \frac{\alpha_j}{\sigma^*_j} \right) L_g - \beta r L \right) - (1 - E^{\gamma=0}) u(L_g - r L) - E^{\gamma=0} u\left(1 - \frac{\alpha_j}{\sigma^*_j} \right) L_g - \beta r L \right) \tag{14}
\]

**Definition 4:** The expected utility of the debtor country when the IMF considers both safeguarding resources and country welfare \((\gamma \in (0,1])\) is defined as follows:

\[
EU^{\gamma} = E^{\gamma} = u\left(1 - \frac{\alpha_j}{\sigma^*_j} \right) L_g - \beta r L \right) + (1 - E^{\gamma}) u\left(1 - \frac{\alpha_j}{\sigma^*_j} \right) L_g - \beta r L \right) - (1 - E^{\gamma}) u(L_g - r L) - E^{\gamma} u\left(1 - \frac{\alpha_j}{\sigma^*_j} \right) L_g - \beta r L \right) \tag{15}
\]

The following proposition states that the IMF being concerned about the debtor country’s welfare does not necessarily force the debtor country to be idle with regard to its policy actions.
Proposition 4: If \( EU^{\gamma = 0} > (\leq) EU^{\gamma} \), the optimal level of policy effort satisfies 
\[ e^{\gamma = 0} > (\leq) e^{\gamma}. \]

Consider the case where the IMF considers the country's welfare (\( \gamma \neq 0 \)). If the difference level in the expected utility of the debtor country corresponding to the good and bad outcomes is smaller, the debtor country makes less effort to overcome the crisis. This implies that mandating the IMF to care about both the country’s welfare and safeguarding its resources does not necessarily mean that the debtor country will exert less effort. The following proposition yields that when \( \gamma \) is not zero, depending on \( x \), the amount of resources available to the IMF, the optimal contract can vary.

Proposition 5: When \( x \to \infty \), the intervention level (\( \sigma \) and \( \sigma_F \)) approaches its maximum. When \( x \to 0 \), the intervention level approaches its minimum.

The amount of resources available to the IMF is important. When the resources available to the IMF are a plenty, the disutility from the intervention cost will be very small. However, the utility of the debtor country will increase significantly through the help of the IMF, and in turn, will become the IMF’s utility depending on \( \gamma \).

On the other hand, when the resources available to the IMF are limited, the utility of the IMF increases substantially through the reduction in the intervention cost given the concavity of the utility function.

4 CONCLUSION

This paper tries to derive the optimal contract between the IMF and the debtor country in the presence of a moral hazard problem and signal noise. The main findings concerning the policy efforts and the noise are as follows. First, we find that when the net loss between an accurate signal and noise under a good fundamental is higher than the net gain under a bad fundamental, the debtor country exerts less effort as the noise effect becomes larger. Therefore, the IMF should minimize the noise so that the country becomes more willing to take large policy actions. Second, when the net loss under a good fundamental is smaller than the net gain under a bad fundamental, an accurate signal may give rise to the moral hazard problem. Intentional concealing of the information may have positive effects to solve this problem. Third, when the marginal utility from the intervention of the IMF is higher under a bad fundamental than under a good fundamental, the higher ability of the IMF to mitigate the crisis will elicit a weaker policy effort from the country. On the other hand, when the economy has higher marginal utility under a good fundamental, deeper intervention of the IMF offers an incentive of a stronger policy effort to the country. The deeper intervention
mitigates the shock, and hence, the country is induced to provide a larger policy response. The main finding about the optimal contract under noise is as follows: mandating the IMF to care about the country’s welfare as well as safeguarding its resources does not necessarily mean that the debtor country will exert less effort.

It is arguable that in this paper the action of the investors is too simplistic. In line with Morris and Shin (1998, 2002), we can specify the strategic interaction among investors. Furthermore, the model is basically divided into three time periods: before the shock, the interim period, and the final period. It would be interesting to extend the timing of intervention more dynamically, which can reduce the loss originating from an incorrect signal. Our model theoretically analyzes the EWS of the IMF, but a more empirical application is necessary in the case of currency crises. Not only domestic problems but also global crises should be considered. Although this paper limits the functions of the IMF, it could be interesting to extend the model to include developing countries under the current global crisis.

Appendix A: Proof for Proposition 1

From (7), \( \frac{\partial e^*}{\partial \varepsilon} \) equals
\[
\frac{\theta^* u[(1 - \frac{\alpha_1}{\sigma_f})L_B - \beta rL] - u[(1 - \frac{\alpha_1}{\sigma_f})L_B - \beta rL] + u(L_G - rL) - u[(1 - \frac{\alpha_3}{\sigma_f})L_B - \beta rL]}{\theta^* u[(1 - \frac{\alpha_1}{\sigma_f})L_B - \beta rL] + (1 - \varepsilon) u[(1 - \frac{\alpha_3}{\sigma_f})L_B - \beta rL] - (1 - \varepsilon) u(L_G - rL) - \varepsilon u[(1 - \frac{\alpha_2}{\sigma_f})L_B - \beta rL]} - v^*.
\]
Since the denominator and \( \theta^* \) are negative, the sign wholly depends on the terms in the bracket in the numerator. Therefore, only if the net loss is bigger than the net benefit,
\[
\left\{ u(L_G - rL) - u\left[\left(1 - \frac{\alpha_3}{\sigma_f}\right)L_B - \beta rL\right] \right\} - \left\{ u\left[\left(1 - \frac{\alpha_2}{\sigma_f}\right)L_B - \beta rL\right] - u\left[\left(1 - \frac{\alpha_3}{\sigma_f}\right)L_B - \beta rL\right]\right\} > 0,
\]
we have that \( \frac{\partial e^*}{\partial \varepsilon} < 0. \)

On the other hand, if
\[
\left\{ u(L_G - rL) - u\left[\left(1 - \frac{\alpha_3}{\sigma_f}\right)L_B - \beta rL\right] \right\} - \left\{ u\left[\left(1 - \frac{\alpha_2}{\sigma_f}\right)L_B - \beta rL\right] - u\left[\left(1 - \frac{\alpha_3}{\sigma_f}\right)L_B - \beta rL\right]\right\} < 0,
\]
we get \( \frac{\partial e^*}{\partial \varepsilon} > 0. \)

Appendix B: Proof for Corollary 1

Q.E.D.
\[
\frac{\partial \theta(e^*)}{\partial \varepsilon} = \frac{\partial \theta(e^*)}{\partial e^*} \frac{\partial e^*}{\partial \varepsilon} > 0.
\]

Appendix C: Proof for Corollary 2

Q.E.D.
\[
\frac{\partial \theta(e^*)}{\partial \varepsilon} = \frac{\partial \theta(e^*)}{\partial e^*} \frac{\partial e^*}{\partial \varepsilon} < 0.
\]
Appendix D: Proof for Proposition 2

From (7), \( \frac{\partial}{\partial \sigma_i} \) equals

\[
\theta' \{ \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_G - \beta r L \} \} \frac{\alpha_i}{\sigma_i} \]

\[
- \theta^* \{ \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} + (1 - \varepsilon) u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} - (1 - \varepsilon) u \{ L_G - r L \} - \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_G - \beta r L \} \} - \nu^* 
\]

Since \( \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} \frac{\alpha_i}{\sigma_i} \) is always positive, \( \frac{\partial}{\partial \sigma_i} < 0 \).  
Q.E.D.

Appendix E: Proof for Proposition 3

From (7), \( \frac{\partial}{\partial \sigma_i} \) equals

\[
\theta' \{ (1 - \varepsilon) u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} \frac{\alpha_i}{\sigma_i} \} - \theta^* \{ \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} + (1 - \varepsilon) u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} - (1 - \varepsilon) u \{ L_G - r L \} - \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_G - \beta r L \} \} - \nu^* 
\]

Therefore, if

\[
\frac{\partial}{\partial \sigma_i} < 0 \{ (1 - \varepsilon) u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} \frac{\alpha_i}{\sigma_i} \} > \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_G - \beta r L \} \frac{\alpha_i}{\sigma_i}
\]

If

\[
\frac{\partial}{\partial \sigma_i} > 0 \{ (1 - \varepsilon) u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} \frac{\alpha_i}{\sigma_i} \} < \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_G - \beta r L \} \frac{\alpha_i}{\sigma_i}
\]

Q.E.D.

Appendix F: Proof for Proposition 4

Rearranging the first-order condition of the debtor country, equation (7), we get

\[
- \frac{\nu'(e)}{\theta'(e)} = (1 - \varepsilon) u \{ L_G - r L \} + \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_G - \beta r L \} \\
- \alpha u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} + (1 - \varepsilon) u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \}
\]

Defining \( \frac{\nu'(e)}{\theta'(e)} = f(e) \), we can get \( f(e) = \frac{f(e) \theta'(e) - \theta'(e) \nu'(e)}{\theta'(e)^2} > 0 \).

By confirming that \( f(e) \) is a strictly increasing function of effort, we can derive Proposition 4.  
Q.E.D.

Appendix G: Proof for Proposition 5

Suppose \( x \to \infty \). Then, the first-order conditions for the IMF become

\[
0 = (1 - \theta) \langle \gamma \frac{\partial u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_G - \beta r L \} \frac{\alpha_i}{\sigma_i}}{\partial \sigma_i} \rangle - \theta (1 - \varepsilon) \left\{ - \gamma \frac{\partial u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_B - \beta r L \} \frac{\alpha_i}{\sigma_i}}{\partial \sigma_i} \right\}
\]

\[
0 = - \theta \langle - \gamma \frac{\partial u \{ (1 - \frac{\alpha_i}{\sigma_i}) L_G - \beta r L \} \frac{\alpha_i}{\sigma_i}}{\partial \sigma_f} \rangle
\]
since the final term in each equation converges to zero and \( \frac{\partial z}{\partial \sigma} \to 0 \). The right-hand side is positive and not close to zero. Therefore, for the equations to be equal to zero, \( \sigma \to \infty \). Q.E.D.

Suppose \( x \to 0 \). Then, the first terms of equations (12) and (13) decrease, and are close to \( \infty \). Therefore, for the equations to be equal to zero, the second term should be close to \( \infty \). Q.E.D.

**Reference**


Review 92 (5):1521–34
