THE CHAOTIC AGRICULTURAL MONOPOLY PROFIT GROWTH MODEL AND INDIRECT TAX

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Abstract

The basic aim of this paper is to construct a relatively simple chaotic growth model of the agricultural monopoly profit that is capable of generating stable equilibria, cycles, or chaos. A key hypothesis of this work is based on the idea that the coefficient \( \mu = f\left[\frac{n - b (1 + d)}{1 + d}\right] \) plays a crucial role in explaining local stability of the agricultural monopoly profit, where, \( b \) – the coefficient of the total cost function of the agricultural monopoly firm, \( n \) - the coefficient of the inverse demand function, \( d \) - an indirect tax rate, \( f \) - the coefficient.

Keywords: agricultural monopoly, profit, chaos

Introduction

Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.

Chaos theory can explain effectively unpredictable economic long time behavior arising in a deterministic dynamical system because of sensitivity to initial conditions. A deterministic dynamical system is perfectly predictable given perfect knowledge of the initial condition, and is in practice always predictable in the short term. The key to long-term unpredictability is a property known as sensitivity to (or sensitive dependence on) initial conditions.


The basic aim of this paper is to provide a relatively simple chaotic agricultural monopoly profit growth model that is capable of generating stable equilibria, cycles, or chaos.

A Simple Chaotic Profit Growth Model of a Profit-Maximizing Agricultural Monopoly

In the model of a profit-maximizing agricultural monopoly, take the inverse demand function
\[ P = n - m Q \]  

(1)

Where \( P \) - agricultural monopoly price; \( Q \) – agricultural monopoly output; \( n, m \) – coefficients of the inverse demand function.

It is supposed that an indirect tax, \( T \), increases the price of an agricultural good so that consumers are actually paying the tax by paying more for the agricultural products, i.e.,

\[ T_t = d P_t \]  

(2)

where \( T \)- an indirect tax, \( d \) – an indirect tax rate, \( P \)- agricultural monopoly price.

In this case,

\[ (1+d) P_t = n - m Q_t \]  

(3)

or

\[ P_t = \frac{n}{1+d} - \frac{m}{1+d} Q_t \]  

(4)

In this case, total revenue, \( TR_t \), is given by

\[ TR_t = P_t Q_t = \frac{n}{1+d} Q_t - \frac{m}{1+d} Q_t^2 \]  

(5)

Further, suppose the quadratic total-cost function for an agricultural monopoly is

\[ TC_t = a + b Q_t + c Q_t^2 \]  

(6)

\( TC \) – marginal cost; \( Q \) – agricultural monopoly output ; \( a, b, c \) – coefficients of the quadratic marginal-cost function.

Profit, \( \Pi \), is the difference between total revenue and total cost. It is supposed that \( a=0 \). Then ,

\[ \Pi_t = \left[ \frac{n-b(1+d)}{1+d} \right] Q_t - \left[ \frac{m+c(1+d)}{1+d} \right] Q_t^2 \]  

(7)

Further, it is supposed that

\[ Q_t = f \Pi_{t-1} \]  

(8)
where: Q – agricultural monopoly output, \( \Pi \) – agricultural monopoly profit, \( f \) - the coefficient.

By substitution one derives:

\[
\Pi_t = f \left[ \frac{n - b (1 + d)}{1 + d} \right] \Pi_{t-1} - f^2 \left[ \frac{m + c (1 + d)}{1 + d} \right] \Pi_{t-1}^2
\] (9)

Further, it is assumed that the agricultural monopoly profit is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the agricultural monopoly profit growth rate depends on the current size of the monopoly profit, \( \Pi \), relative to its maximal size in its time series \( \Pi^m \). We introduce \( \pi \) as \( \pi = \frac{\Pi}{\Pi^m} \). Thus \( \pi \) range between 0 and 1. Again we index \( \pi \) by \( t \), i.e., write \( \pi_t \) to refer to the size at time steps \( t = 0,1,2,3,... \) Now, growth rate of the agricultural monopoly profit is measured as

\[
\pi_t = f \left[ \frac{n - b (1 + d)}{1 + d} \right] \pi_{t-1} - f^2 \left[ \frac{m + c (1 + d)}{1 + d} \right] \pi_{t-1}^2
\] (10)

This model given by equation (10) is called the logistic model. For most choices of \( b, c, d, m, n, \) and \( f \) there is no explicit solution for (10). Namely, knowing \( b, c, d, m, n, \) and \( f \) and measuring \( \pi_0 \) would not suffice to predict \( \pi_t \) for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (10) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of \( \pi_t \). This difference equation (10) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point \( \pi_0 \) the solution is highly sensitive to variations of the parameters \( b, c, d, m, n, \) and \( f \); secondly, given the parameters \( b, c, d, m, n, \) and \( f \) the solution is highly sensitive to variations of the initial point \( \pi_0 \). In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

**Logistic Equation**

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation

\[
z_{t+1} = \mu z_t (1 - z_t), \quad \mu \in [0,4], \quad z_t \in [0,1]
\] (11)
is equivalent to the iteration of growth model (10) when we use the following identification:

\[ z_t = \frac{m+c(1+d)}{n-b(1+d)} \pi_t \quad \text{and} \quad \mu = f \left[ \frac{n-b(1+d)}{1+d} \right] \]

(12)

Using (10) and (12) we obtain

\[ z_{t+1} = \frac{m+c(1+d)}{n-b(1+d)} \pi_{t+1} = \frac{m+c(1+d)}{n-b(1+d)} \left\{ f \left[ \frac{n-b(1+d)}{1+d} \right] \pi_t - f^2 \left[ \frac{m+c(1+d)}{1+d} \right] \pi_t^2 \right\} = \]

\[ = \frac{f^2[m+c(1+d)]}{1+d} \pi_t - \frac{f^3[m+c(1+d)]^2}{n(1+d)-b(1+d)^2} \pi_t^2 \]

On the other hand, using (10), (11), and (12) we obtain

\[ z_{t+1} = \mu z_t (1 - z_t) = f \left[ \frac{n-b(1+d)}{1+d} \right] f \left[ \frac{m+c(1+d)}{n-b(1+d)} \right] \pi_t \left\{ 1 - f \left[ \frac{m+c(1+d)}{n-b(1+d)} \right] \pi_t \} \right\} = \]

\[ = \frac{f^2[m+c(1+d)]}{1+d} \pi_t - \frac{f^3[m+c(1+d)]^2}{n(1+d)-b(1+d)^2} \pi_t^2 \]

Thus we have that iterating \( \pi_t = f \left[ \frac{n-b(1+d)}{1+d} \right] \pi_{t-1} - f^2 \left[ \frac{m+c(1+d)}{1+d} \right] \pi_{t-1}^2 \)

is really the same as iterating \( z_{t+1} = \mu z_t (1 - z_t) z_t = f \left[ \frac{m+c(1+d)}{n-b(1+d)} \right] \pi_t \) and \( \mu \)

\[ = f \left[ \frac{n-b(1+d)}{1+d} \right] \] (see Figure 1.)

It is important because the dynamic properties of the logistic equation (11) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that:

(i) For parameter values \( 0 < \mu < 1 \) all solutions will converge to \( z = 0 \);
(ii) For \( 1 < \mu < 3.57 \) there exist fixed points the number of which depends on \( \mu \);
(iii) For \( 1 < \mu < 2 \) all solutions monotonically increase to \( z = (\mu - 1) / \mu \);
(iv) For \( 2 < \mu < 3 \) fluctuations will converge to \( z = (\mu - 1) / \mu \);
(v) For $3 < \mu < 4$ all solutions will continuously fluctuate;
(vi) For $3.57 < \mu < 4$ the solutions become "chaotic" which means that there exist totally aperiodic solutions or periodic solutions with a very large, complicated period. This means that the path of $z_t$ fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

\[
\pi_t = f \left[ \frac{n - b (1 + d)}{1 + d} \right]
\]

plays a crucial role in explaining local stability of the agricultural monopoly profit, where, $b$ - the coefficient of the total cost function of the agricultural monopoly firm, $n$ - the coefficient of the inverse demand function, $d$ - an indirect tax rate, $f$ - the coefficient.
References
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